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OFFICE NOTE 248

A Note on the Use of Lower Saturation Mixing Ratio Criteria for Release of Latent Heat in NWP Models

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1. Introduction

The large scale release of latent heat (isobaric condensation of water vapor) in most numerical models is evaluated when the mixing ratio $q(T,p)$ exceeds $q_{sm}(T,p)$, where

$$q_{sm}(T,p) = SATRH \; q_s(T,p) \; . \tag{1}$$

Here $q_s(T,p)$ is the saturated mixing ratio at the grid point with temperature T and pressure p. The case SATRH=1 is referred to below as unmodified case (simulates condensation at a supersaturated point in the real atmosphere) and case SATRH<1 as modified case. Note that in the modified case, use of a fixed value of SATRH, say 0.8, implies that at the end of any time step $q(T,p) \leq 0.8\, q_s(T,p)$. The relative humidity (RH) in such a model cannot attain values greater than 80%. Values of SATRH ranging from 0.8 to 0.96 are used in NMC's operational models (e.g., see Gerrity, 1976 and Phillips, 1979).

The vertical profiles of T and q for which cooling due to vertical advective term $-\omega \frac{\partial \theta}{\partial p}$ is balanced by the large scale release of latent heat (supersaturation arising from the upward advection of moisture $-\omega \frac{\partial q}{\partial p}$) for modified and unmodified cases are presented in Section 2. Here ω is vertical p velocity. The profiles for the unmodified cases are given by pseudo-adiabats. The profile in the modified case is found to be significantly colder than the corresponding profile in the unmodified case.

In order to show the impact of the use of a fractional value of SATRH in numerical models, the results from the time integrations of NMC's 2D hurricane model in the modified and unmodified cases are compared in Section 3. The initial disturbance intensifies into severe hurricane in the unmodified case and nearly pseudo-adiabatic profile is attained near the center. The intensity of the simulated disturbance is much weaker in the modified case (SATRH=.8) and the vertical potential temperature profile attained near the center is nearly the same as derived for the modified case in Section 2.

2. Condensation due to large scale upward motion in the model's saturated environment

a. Unmodified case

Consider the case when the lapse rate at a grid point A is such that $T_e(p)=T_c(p)$, and $q_e(p)=q_c(p)$. Here, subscript e denotes the value at the grid point and c the value along a pseudo-adiabat through the lowest information level. The environment at the grid point is therefore saturated with the convective clouds. Since this state of the atmosphere is moist adiabatically neutral state, Kuo-type convective parameterization scheme (or convective adjustment type scheme like one used in NMC's LFM model) would give no change in this profile.

Now consider the changes in (q_e,T_e) at the grid point A, due to large scale upward motion (i.e., changes associated with the vertical advective term in (x,y,p) coordinate system and any isobaric condensation resulting from it).

$$\frac{\partial q_e}{\partial t} = -\omega\frac{\partial q_e}{\partial p} = -\omega\frac{\partial q_c}{\partial p}$$

This increase in moisture (q_c for a pseudo-adiabatic lapse rate decreases with p) would give rise to supersaturation. If all the excess moisture is condensed so that

$$\frac{\partial q_e}{\partial t} = 0 \quad , \tag{2}$$

and

$$\frac{\partial \theta_e}{\partial t} = -\omega\frac{\partial \theta_c}{\partial p} - L\omega\frac{\theta_c}{c_p T_c}\frac{\partial q_c}{\partial p} \quad . \tag{3}$$

Since the last term represents the latent heat released in an isobaric condensation of water vapor and because the equivalent potential temperature $\theta_E \simeq \theta_c + \frac{Lq_c}{c_p}\frac{\theta_c}{T_c}$ is constant along a pseudo-adiabat, we may rewrite the above equation as

$$\frac{\partial \theta_e}{\partial t} = -\omega\frac{\partial}{\partial p}\left[\theta_c + \left(\frac{\theta_c}{T_c}\right)\frac{Lq_c}{c_p}\right] = 0 \quad . \tag{4}$$

Here, L is latent heat of condensation and c_p is specific heat of air at constant pressure.

We now summarize the above results. Consider the case when a pseudo-adiabatic lapse rate develops at a grid point (A) at time=t in a numerical model. The temperature and mixing ratio at A are the same as along a pseudo-abiabat through the lowest level's temperature. No change in this lapse curve at A would occur in the model's next time step due to moist convection, if a Kuo-type convective parameterization procedure (or NMC's LFM type convective adjustment procedure) is used in the model. Furthermore, if the large scale release of latent heat is evaluated when q_e exceeds the saturation value q_s (SATRH=1, unmodified case) then according to Eqs. (2) and (4), the pseudo-adiabatic lapse curve at A would not change due to large scale upward motion. The cooling due to vertical advection term $-\omega \frac{\partial \theta}{\partial p}$ in the model is balanced by the large scale release of latent heat (supersaturation associated with upward advection of moisture $-\omega \frac{\partial q}{\partial p}$). Therefore the lapse curve (5) is unchanged with respect to convective as well as large scale upward motions.

$$\theta_E \simeq \theta_c + \frac{L q_c}{c_p} \frac{\theta_c}{T_c} = \text{constant} . \tag{5}$$

b. Modified case

Consider the case when the (vertical) lapse rate at a grid point B satisfies Eq. (6)

$$\theta_c' + \frac{L q_c'}{c_p} \frac{\theta_c'}{T_c'} = \text{constant} \tag{6}$$

where

$$q_c' = q_{sm}(T_c', p) = \text{SATRH}\; q_s(T_c', p) . \tag{7}$$

Now consider the changes in (q_e, T_e) due to large scale upward motion at the grid point B.

$$\frac{\partial q_e}{\partial t} = -\omega \frac{\partial q_e}{\partial p} = -\omega \frac{\partial q_c'}{\partial p} \qquad (8)$$

For all values of SATRH<1, $T_c'(p)<T_c(p)$. (The curve (6) for a case SATRH=.8 is shown in Figure 1.) Furthermore q_c' decreases with p. Therefore the increase in moisture due to vertical advective term would give rise to $q_e>q_{sm}(T_c',p)$. If the excess moisture is condensed, so that $\frac{\partial q_e}{\partial t} = 0$, we have (in view of Eq. (6))

$$\frac{\partial \theta_e}{\partial t} = -\omega \frac{\partial \theta_c'}{\partial p} - \frac{L\omega}{c_p} \frac{\partial q_c'}{\partial p} \frac{\theta_c'}{T_c'} = 0$$

Therefore the lapse curve (6) is unchanged with respect to large scale upward motion in the modified case.

Most convective parameterization schemes used in numerical models, tend to adjust the environmental temperature and moisture vertical lapse profiles towards that of a model convective cloud represented by the pseudo-adiabatic profile (Eq. 5). A numerical example given in the appendix shows that use of model convective cloud lapse rate given by Eq. (5) in the modified case could give rise to convectively adjusted mixing ratio q_e much larger (unrealistically) than q_{sm}. If isobaric condensational process is invoked to adjust q_e to q_{sm}, unrealistically large heating of the environment would be simulated in the model. This difficulty does not arise if the convective model cloud is represented by the vertical profile given by Eq. (6) in the modified case.

c. Comparison between unmodified and modified cases

Consider the case where the lapse rate given by Eq. (5) exists at the grid point A in the unmodified case. According to our discussion in Section (2a), this lapse rate is unchanged with respect to the convective as well as large scale upward motions.

Let the model convective cloud lapse rate be given by Eq. (6) in the modified case. Then according to our discussion in Section 2b, this lapse rate is unchanged with respect to convective as well as large scale upward motions in the model.

The lapse rate curve, given by Eq. (5) and (6) are shown in Figure 1 for temperature of 20C at 1000 mb. Note that temperatures above 500 mb along curve marked B (modified case Eq. (6), SATRH=0.8) are much colder than those along corresponding curve marked A (unmodified case Eq. (5)). The above differences would decrease if a higher value of SATRH is used.

Consider the case when a unit vertical column above a grid point is predicted to be saturated with convective clouds during the time integration of a numerical model. The lapse curve in this vertical column would be given by Eq. (5) for unmodified case and Eq. (6) for the modified case. The vertical column will be much warmer in the unmodified case compared to that in the modified case. Since the lapse rate in a column saturated with the convective clouds in the real atmosphere is nearly given by Eq. (5), the above discussion implies that in general the net heating in a vertical conditionally unstable column is underpredicted in the numerical model which uses values of SATRH less than unity.

3. Numerical results

The vertical temperature profile in the wall clouds near the center of hurricanes is observed to nearly coincide with the pseudo-adiabatic lapse curve through the temperature and pressure point at the base of these clouds. This structure of the wall cloud is well simulated in axially symmetric hurricane models when integrated to nearly steady state.

The NMC's 2D hurricane model is used in this study. This model uses σ as the vertical coordinate and ten vertical layers. The horizontal grid distance is 60 km with 22 equidistant points in the radial direction. In the operational cycle the 2D model is integrated on f plane (coriolis parameter corresponding to the latitude of the observed storm) to nearly steady state solution. Value of SATRH=.8 is used and sea surface temperature is fixed (usually at 301.2 k) in time and space. Air-sea exchange of sensible and latent heat and the Kuo-type convective parameterization procedure are incorporated in the model. The initial state consists of a symmetric vortex (maximum tangentral winds of about 30 m s^{-1}). The vortex simulated by this 2D model is superimposed as a perturbation on the operationally (Hough) analyzed fields. The above procedure prescribes the initial data for the time integration of NMC's 3D hurricane model MFM (Moving Fine Mesh model).

We now present the results of integration of 2D hurricane model for unmodified (SATRH=1) and modified (SATRH=.8) cases. The value of f at 25N is used in these experiments. A quasi-steady state is attained in both experiments after 50 hours. The variation of maximum tangential velocity ($V_{\theta\,max}$) with time in two cases is shown in Fig. 2. The maximum winds are located in the lower troposphere close to the center. The maximum winds near the center at 60 hours are less than 36 m s^{-1} in the modified case while they attain values close to 60 m s^{-1} in the unmodified case. The lapse rate at the grid point next to center lies close to the pseudo-adiabat through the lowest level temperature in the unmodified case (curve A in Figure 3); it is much colder in the modified case (curve B in Figure 3).

4. Conclusions

Most parameterization schemes used in the numerical models to simulate convective release of latent heat realistically tend to adjust the conditionally unstable regions in the model towards the moist adiabatically netural state given by the pseudo-adiabat through the base of the convective cloud. This neutral lapse rate can be simulated in the model only if the isobaric condensation of water vapor takes place when the mixing ratio exceeds the saturation value at the grid point (unmodified case, SATRH=1). This is because only in this case, given the moist adiabatic lapse rate at the grid point, the cooling due to the vertical advective term $-\omega \frac{\partial \theta}{\partial p}$ is balanced by the isobaric condensational release of latent heat; supersaturation arising due to upward vertical advection of moisture ($-\omega \frac{\partial q}{\partial p}$ term). It is worth noting that in order to simulate the adjustment of conditionally unstable atmosphere towards moist neutral state realistically, the large scale release of latent heat in the conditionally unstable regions must be included in the model. The large scale release of latent heat at conditionally unstable grid points has been omitted in some previous works (e.g., Mathur, 1975, Krishnamurti, 1973).

Thermodynamic considerations presented in Section 2 and numerical results presented in Section 3 show that when the large scale release of latent heat is evaluated in the model at less than 100% saturation (modified case, SATRH<1), the conditionally unstable regions are adjusted towards a modified vertical lapse curve (curves B in Figures 1 and 3). The temperatures along these modified curves are much colder than those along the corresponding pseudo-adiabats through the base of the convective clouds. Therefore, the net heating of the atmosphere is much weaker in modified case compared to that in unmodified case. The warm core of hurricanes can apparently be only simulated if SATRH is assigned value close to unity.

The use of SATRH<1 in the models is based on the premise that the value of a variable at the grid point represents an average value of the variable in a three-dimensional box centered at the grid point. The isobaric condensational release of latent heat simulates the formation of layered clouds in the atmosphere. The layered clouds in the real atmosphere would, in general, occupy only a fractional surface area of the grid box at any time. Therefore even though the relative humidity RH in the clouds is 100%, the average value of RH over the grid box in the real atmosphere (with only fraction of the grid box area occupied with layered cloud) would be less than 100%. Therefore the large scale release of latent heat in a numerical model may be invoked at the value of RH less than 100%. A simple scheme which allows the RH at the model grid point to attain value of 100% (case when grid box is completely covered with clouds) and also evaluates large scale release of latent heat when the grid box is partially covered with clouds (RH>80%) was formulated. The structure of the simulated hurricane using NMC's 2D hurricane model and the above scheme is similar to that obtained in the unmodified case. The formulation of this scheme and the numerical results with 2D hurricane model and MFM are presented in a subsequent report.

APPENDIX

A numerical example of unrealistic release of latent heat in the numerical models in the modified case.

We consider Kuo type convective parameterization procedure. In this procedure we first determine the fraction (α) of unit area surrounding the grid point, which is covered by newly formed convective clouds.

$$\alpha = \frac{I\,\Delta t}{Q},$$

where Δt is time step, I is rate of total moisture convergence in a unit column extending from base of the cloud to the top of the cloud. Q is the total moisture needed to saturate the entire vertical column between the base and top of the cloud. To parameterize the effects of convection on the environment, it is assumed that the cloudy air mixes with the environment at the same level. The adjusted temperature (T_F) and mixing ratio (q_F) are given respectively by

$$T_F = T_e + \alpha (T_c - T_e) \tag{A1}$$

provided $T_c > T_e$, otherwise $T_F = T_e$, and

$$q_F = q_e + \alpha (q_c - q_e) \tag{A2}$$

provided $q_c > q_e$ otherwise $q_F = q_e$.

We now present an example of convective parameterization procedure in unmodified and modified cases to show that unrealistic warming may result in the modified case if T_c and q_c satisfy Eq. (5). We consider a somewhat large value of α to exaggerate the results.

Let $\alpha = 0.1$, $p = 800$ mb, $T_c = 293.66$K
$T_e = 293.16$, and $q_e = q_{sm}\,(T_e, p)$

The computations were performed on IBM 360/195 computer.

a. Unmodified case (SATRH=1)

$$q_e = q_s\ (T_e, p) = .01837$$

$$q_c = q_s\ (T_c, p) = .01895$$

From (A1) and (A2) we obtain

$$T_F = 293.21\text{K},\ q_F = .01843$$

Since $q_s\ (T_F, p) = .01843$, the adjusted state is saturated.[1]

b. Modified case (SATRH=0.8)

(q_c, T_c) satisfy Eq. 5.

$$q_e = 0.8\ q_s\ (T_e, p) = .01469$$

From (A1) and (A2) we obtain

$$T_F = 293.21\text{K},\ q_F = .01512$$

Now $q_{sm}\ (T_F, p) = .8\ \ q_s\ (T_F, p) = .01474$.

If the excess moisture $q_F - q_{sm} = .00038$ is condensed (the large scale release of latent heat is calculated after convective parameterization is completed as is the case in NMC's 2D hurricane model and the MFM), we would obtain the final adjusted environmental temperature T_F' at 294.15K (for $\frac{L}{c_p}\ .00038 \simeq .95$) which is greater than T_c. This is not realistic.

In the numerical integration carried out using 2D hurricane model, value of SATRH=0.8 was used in the modified case. The model convective cloud in Kuo parameterization procedure was given by an appropriate pseudo-adiabat. Results of integration (Section 3) show that modified lapse profile B (Figure 3) was obtained near the center in this case. The vertical profile of temperature is therefore mainly determined by value of SATRH used (large scale considerations for the release of latent heat) and not by model convective cloud sounding used

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[1]Very small departures from saturated state may occur in case a and c because mixing ratio q does not vary linearly with temperature.

in the parameterization procedure. Consider the case when a temperature profile (e.g., a pseudo-adiabat through the base of profile B in Figure 3) warmer than the profile given by Eq. 6 develops in the modified case. For such a warmer profile, the cooling due to vertical advection term $-\omega \frac{\partial \theta}{\partial p}$ will be larger than heating due to large scale release of latent heat (super-saturation occurring due to vertical advection of moisture). Even if the environment were saturated with mixing ratio at $q_{sm}(T,p)$, the term $\left| -\omega \frac{\partial \theta}{\partial p} \right|$ is larger than $\left| -\frac{L}{c_p} \frac{\theta}{T} \omega \frac{\partial q_{sm}}{\partial p} \right|$. The environment will cool till profile given by Eq. (6) is obtained.

c. Modified case (SATRH=0.8)

$T_c' = T_c = 293.66K$ but (q_c', T_c') satisfy Eq. (6)

$$q_c' = .8\ q_s\ (T_c',p) = .01516$$

Then from (A1) and (A2)

$$T_F = 293.21K$$

$$q_F = .01474$$

Since $q_{sm}(T_F,p) = .01474$ there is no supersaturation.

References

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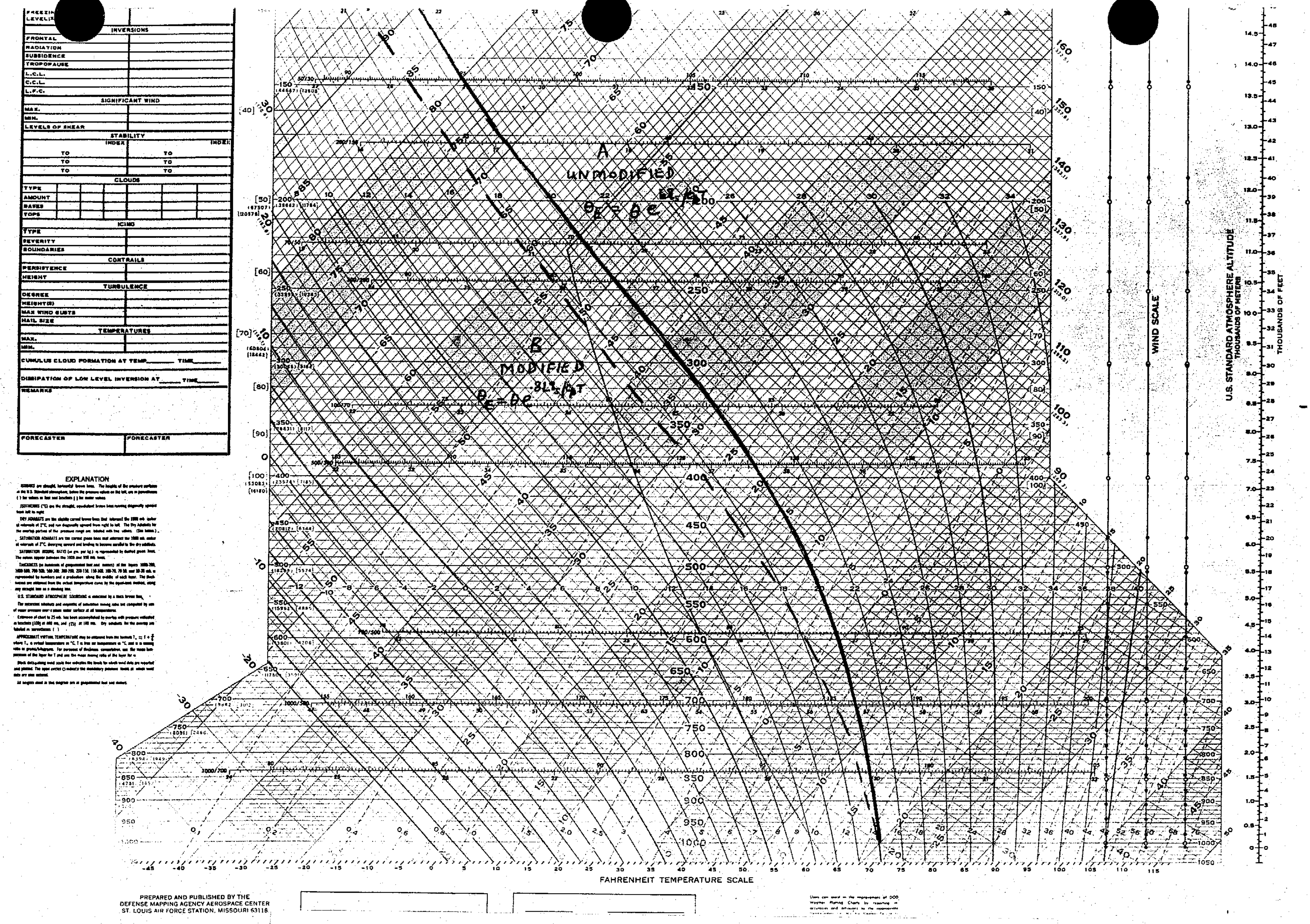

Figure 1. Vertical lapse rate profiles through the temperature of 20C at 1000 mb for (1) unmodified case curve A, $\theta_E = \theta e^{Lq_s/c_pT} \simeq \theta + \frac{L}{c_p} q_s \frac{\theta}{T}$ = constant, and (2) modified case curve B, $\theta_E' = \theta e^{\delta L q_s/c_pT} \simeq \theta + \frac{L}{c_p} \delta q_s \frac{\theta}{T}$ = constant.

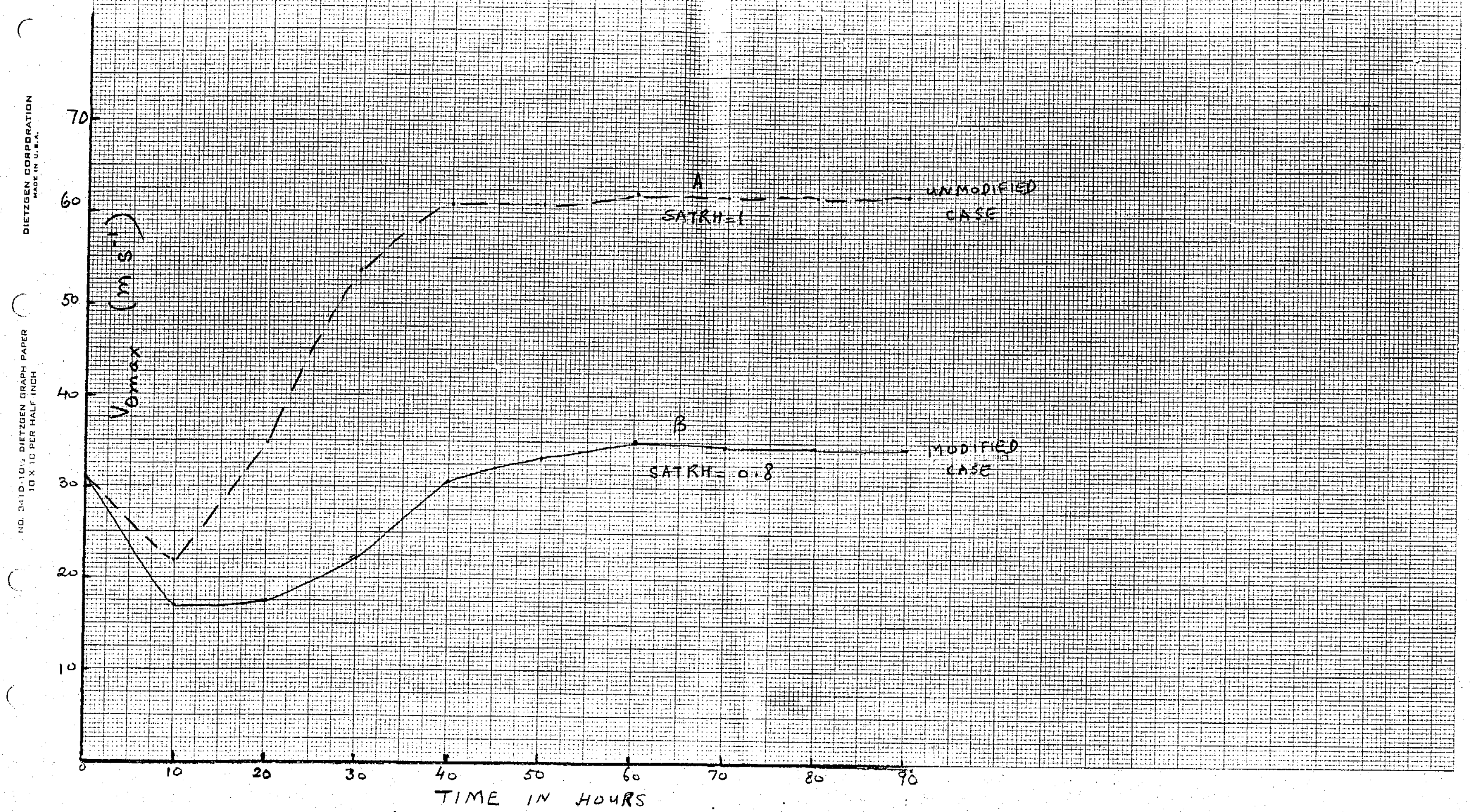

Figure 2. Maximum tangential winds ($V_{\theta max}$) during the time integration of 2D hurricane model: (1) unmodified case curve A, SATRH=1, and (2) modified case, SATRH=.8. Note that maximum winds are of the order of 60 m s^{-1} at 60 hours in unmodified case while they are less than 36 m s^{-1} in the modified case.

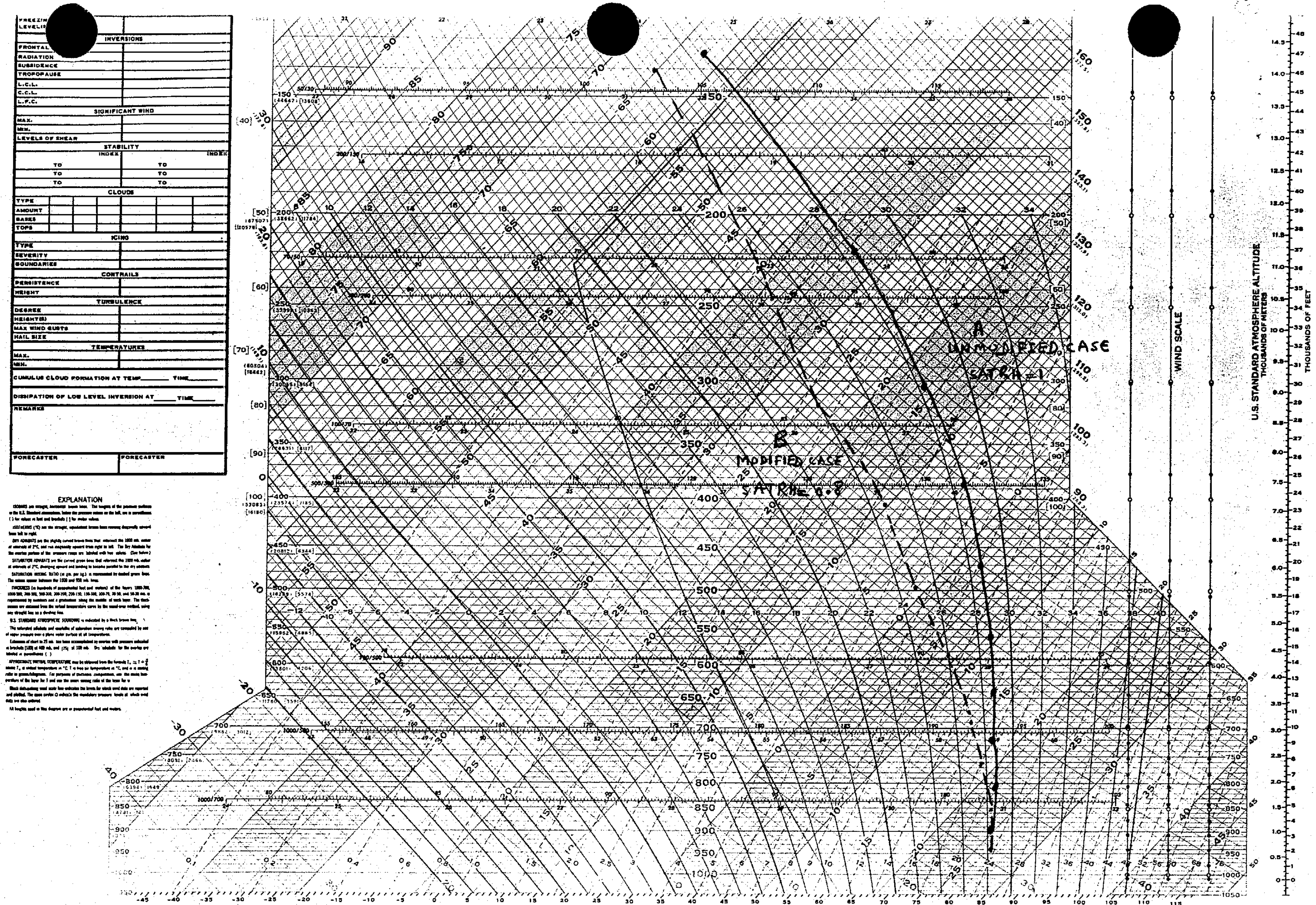

Figure 3. Vertical temperature profiles near the center of simulated symmetric hurricanes at 60 hr: (1) unmodified case curve A, SATRH=1, and (2) modified case, curve B, SATRH=.8. Note that profile A nearly coincides with a pseudo-adiabat and profile B is much colder than profile A.